Geometry, Optimization and Control in Robot Coordination

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Distributed Control of Robotic Networks

1. intro to distributed algorithms (graph theory, synchronous networks, and averaging algos)
2. geometric models and geometric optimization problems
3. model for robotic, relative sensing networks, and complexity
4. algorithms for rendezvous, deployment, boundary estimation

## Cooperative multi-agent systems

**What kind of systems?**
Groups of agents with control, sensing, communication and computing

**What kind of abilities?**
- each agent **senses** its immediate environment,
- **communicates** with others,
- **processes** information gathered, and
- **takes local action** in response

<table>
<thead>
<tr>
<th>Image 1</th>
<th>Image 2</th>
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</thead>
<tbody>
<tr>
<td><img src="image1" alt="AeroVironment Inc, “Raven”" /></td>
<td><img src="image2" alt="iRobot Inc, “PackBot”" /></td>
</tr>
<tr>
<td>unmanned aerial vehicle</td>
<td>unmanned ground vehicle</td>
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</tbody>
</table>

AeroVironment Inc, “Raven”
iRobot Inc, “PackBot”
Cooperative systems: technologies and applications

What kind of tasks?

Security systems, disaster recovery, environmental monitoring, study of natural phenomena and biological species, science imaging

What scenarios?

Security systems, disaster recovery, environmental monitoring, study of natural phenomena and biological species, science imaging
Today's outline

1. **vehicle routing problems**
   via queueing theory and combinatorics

2. **territory partitioning**
   via emerging behaviors and geometric optimization

3. **peer-to-peer coordination**
   via invariance principle on metric space
Queueing theory for robotic networks

Dynamic Vehicle Routing
- customers appear randomly space/time
- robotic network knows locations and provides service
- Goal: distributed adaptive algos, delay vs throughput

Algo #1: Receding-horizon shortest-path policy

Receding-horizon Shortest-Path (RH-SP)

For $\eta \in (0, 1]$, single agent performs:
1: while no customers, move to center
2: while customers waiting
   1. compute shortest path through current customers
   2. service $\eta$-fraction of path

- shortest path is NP-hard, but effective heuristics available
- delay is optimal in light traffic
- delay is constant-factor optimal in high traffic
Algo #1: Sketch of RH-SP analysis 
via combinatorics in Euclidean space 

queue is stable if \( \text{service time} < \text{interarrival time} \)

\[
\text{service time} = \frac{\text{length shortest path}(n)}{n} \quad (n = \# \text{ customers})
\]

queue is stable if \( \text{length of shortest path} = \text{sublinear } f(n) \)

\[ \text{length shortest path}(n) \sim \sqrt{n} \]

Algo #2: Load balancing via territory partitioning

**RH-SP + Partitioning**

For $\eta \in (0, 1]$, agent $i$ performs:

1. compute own cell $v_i$ in optimal partition
2. apply RH-SP policy on $v_i$

Asymptotically constant-factor optimal in light and high traffic
1. vehicle routing problems
   via queueing theory and combinatorics

2. territory partitioning
   via emerging behaviors and geometric optimization

3. peer-to-peer coordination
   via invariance principle on metric space
Territory partitioning akin to *animal territory dynamics*

- Tilapia mossambica, “Hexagonal Territories,” Barlow et al, ’74
- Sage sparrows, “Territory dynamics in a sage sparrows population,” Petersen et al ’87
Territory partitioning: behaviors and optimality

ANALYSIS of cooperative distributed behaviors

1. how do animals share territory?
   how do they decide foraging ranges?
   how do they decide nest locations?
2. what if each robot goes to “center” of own dominance region?
3. what if each robot moves away from closest vehicle?

DESIGN of performance metrics

1. how to cover a region with \( n \) minimum-radius overlapping disks?
2. how to design a minimum-distortion (fixed-rate) vector quantizer?
3. where to place mailboxes in a city / cache servers on the internet?
Multi-center functions

Expected wait time

\[ H(p, v) = \int_{v_1} \| q - p_1 \| dq + \cdots + \int_{v_n} \| q - p_n \| dq \]

- \( n \) robots at \( p = \{p_1, \ldots, p_n\} \)
- environment is partitioned into \( v = \{v_1, \ldots, v_n\} \)

\[ H(p, v) = \sum_{i=1}^{n} \int_{v_i} f(\| q - p_i \|) \phi(q) dq \]

- \( \phi : \mathbb{R}^2 \rightarrow \mathbb{R}_{\geq 0} \) density
- \( f : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R} \) penalty function
The Voronoi partition $\{V_1, \ldots, V_n\}$ generated by points $(p_1, \ldots, p_n)$

$$V_i = \{ q \in Q | \|q - p_i\| \leq \|q - p_j\|, \forall j \neq i \}$$

$$= Q \bigcap_j \text{(half plane between } i \text{ and } j, \text{ containing } i \text{)}$$
Optimal centering (for region $v$ with density $\phi$)

<table>
<thead>
<tr>
<th>function of $p$</th>
<th>minimizer = center</th>
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<tr>
<td>$p \mapsto \int_{v} |q - p|^2 \phi(q) dq$</td>
<td>centroid (or center of mass)</td>
</tr>
<tr>
<td>$p \mapsto \int_{v} |q - p| \phi(q) dq$</td>
<td>Fermat–Weber point (or median)</td>
</tr>
<tr>
<td>$p \mapsto \text{area}(v \cap \text{disk}(p, r))$</td>
<td>$r$-area center</td>
</tr>
<tr>
<td>$p \mapsto \text{radius of largest disk centered at } p$ enclosed inside $v$</td>
<td>incenter</td>
</tr>
<tr>
<td>$p \mapsto \text{radius of smallest disk centered at } p$ enclosing $v$</td>
<td>circumcenter</td>
</tr>
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From online Encyclopedia of Triangle Centers
From optimality conditions to algorithms

\[ H(p, v) = \sum_{i=1}^{n} \int_{V_i} f(||q - p_i||) \phi(q) dq \]

Theorem (Alternating Algorithm, Lloyd '57)

1. at fixed positions, optimal partition is Voronoi
2. at fixed partition, optimal positions are “generalized centers”
3. alternate v-p optimization

\[ \implies \text{local optimum} = \text{center Voronoi partition} \]
Voronoi+centering algorithm

Voronoi+centering law
At each comm round:
1: acquire neighbors’ positions
2: compute own dominance region
3: move towards center of own dominance region

Area-center  
Incenter  
Circumcenter

Experimental Territory Partitioning

Takahide Goto, Takeshi Hatanaka, Masayuki Fujita
Tokyo Institute of Technology
Experimental Territory Partitioning

Optimal Distributed Coverage Control for Multiple Hovering Robots with Downward Facing Cameras

Mac Schwager
Brian Julian
Daniela Rus
Distributed Robots Laboratory, CSAIL

Mac Schwager, Brian Julian, Daniela Rus
Distributed Robots Laboratory, MIT
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| 1. **vehicle routing problems**  
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via emerging behaviors and geometric optimization |
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via invariance principle on metric space |
Partitioning with minimal communication requirements

Voronoi+centering law requires:
1. synchronous communication
2. communication along edges of dual graph

Minimalist coordination
- is synchrony necessary?
- is it sufficient to communicate peer-to-peer (gossip)?
- what are minimal requirements?
Peer-to-peer partitioning policy

1. Random communication between two regions
2. Compute two centers
3. Compute bisector of centers
4. Partition two regions by bisector

P. Frasca, R. Carli, and F. Bullo. Multiagent coverage algorithms with gossip communication: control systems on the space of partitions. In American Control Conference, pages 2228–2235, St. Louis, MO, June 2009
Indoor example implementation

- Player/Stage platform
- realistic robot models in discretized environments
- integrated wireless network model & obstacle-avoidance planner

Peer-to-peer convergence analysis (proof sketch 1/3)

Lyapunov function for peer-to-peer territory partitioning

\[ H(v) = \sum_{i=1}^{n} \int_{v_i} f(\|\text{center}(v_i) - q\|) \phi(q) dq \]

- **state space** is not finite-dimensional
  non-convex disconnected polygons
  arbitrary number of vertices
- peer-to-peer map is not deterministic, ill-defined and discontinuous
  two regions could have same centers
The space of partitions (proof sketch 2/3)

**Definition (space of \(n\)-partitions)**

\(v\) is collections of \(n\) subsets of \(Q, \{v_1, \ldots, v_n\}\), such that

1. \(v_1 \cup \cdots \cup v_n = Q\),
2. \(\text{interior}(v_i) \cap \text{interior}(v_j) = \emptyset\) if \(i \neq j\), and
3. each \(v_i\) is closed, has non-empty interior and zero-measure boundary

Given sets \(A, B\), **symmetric difference** and **distance** are:

\[d_\Delta(A, B) = \text{area}\left((\text{points in } A \text{ that are not in } B) \cup (\text{vice versa})\right)\]

**Theorem (topological properties of the space of partitions)**

Partition space with \((u, v) \mapsto \sum_{i=1}^n d_\Delta(u_i, v_i)\) is metric and precompact
Convergence with persistent switches (proof sketch 3/3)

- X is metric space
- finite collection of maps $T_i : X \to X$ for $i \in I$
- consider sequences $\{x_\ell\}_{\ell \geq 0} \subset X$ with

$$x_{\ell+1} = T_i(\ell)(x_\ell)$$

Assume:

- $W \subset X$ compact and positively invariant for each $T_i$
- $U : W \to \mathbb{R}$ decreasing along each $T_i$
- $U$ and $T_i$ are continuous on $W$
- there exists probability $p \in ]0,1[$ such that, for all indices $i \in I$ and times $\ell$, we have $\text{Prob}[x_{\ell+1} = T_i(\ell)(x_\ell) \mid \text{past}] \geq p$

If $x_0 \in W$, then almost surely

$$x_\ell \to (\text{intersection of sets of fixed points of all } T_i) \cap U^{-1}(c)$$
Emerging discipline: robotic networks

Robotic Network Theory

1. **network modeling**
   - network, ctrl+comm algorithm, task, complexity

2. **coordination algorithm**
   - partitioning, vehicle routing, task allocation

Open problems

1. algorithmic design for minimalist robotic networks
   - scalable, adaptive, asynchronous, agent arrival/departure
   - rich task set, e.g., cooperative estimation

2. mixed robotic-human networks

3. high-fidelity sensing/actuation scenarios